PROCEEDINGS OF THE VIRTUAL MATHEMATICS IN INDUSTRY STUDY GROUP

2021

Virtual Mathematics in Industry Study Group South Africa

VMISGSA 2021

The writing of a Technical Report for the Proceedings of the MISGSA was coordinated by the moderator of the problem. Sections of the Report were written by the moderator and by other members of the study group who worked on the problem.

The Editor of the Proceedings was

Prof D P Mason (University of the Witwatersrand, Johannesburg)

The Technical Reports were submitted to the Editor. Each Report was refereed by one referee. On the recommendation of the referees the Reports were accepted for the Proceedings subject to corrections and minor revisions. The Editor would like to thank the referees for their assistance by refereeing the Reports for the Proceedings.

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PREFACE

The eighteenth Mathematics in Industry Study Group (MISG) in South Africa was held at the University of the Witwatersrand, Johannesburg, from Monday 1 Febuary to Friday 5 Febuary 2021.

The total number of registered participants at the MISG was fifty-nine. There were twenty-three Academic Staff, twenty-nine Graduate Students and seven Industry Representatives. The invited guests were:

Neville Fowkes	University of Western Australia, Australia
Graeme Hocking	Murdoch University, Perth, Australia
Hilary Ockendon	University of Oxford, United Kingdom
John Ockendon	University of Oxford, United Kingdom
Ian Griffiths	University of Oxford, United Kingdom
Tim Myers	Centre de Recerca Matematica, Barcelona, Spain
Sharidan Shafie	University of Technology, Malaysia
Shariffah Suhsila	University of Technology, Malaysia
Dhanesh Patesh	University of Baroda, Vadodara, India
Amit Patesh	University of Baroda, Vadodara, India
Mrudula Trivedi	University of Baroda, Vadodara, India

The South African Universities and Institutes which were represented were:

African Institute for Mathematical Sciences
North-West University
Rhodes University
University of Johannesburg
University of KwaZulu-Natal
University of Mpumalanga
University of South Africa (UNISA)
University of Stellenbosch
University of the Witwatersrand
University of Zululand

A Virtual MISG allows participation of researchers who otherwise would find it difficult to participate in the MISG. This includes experts in the field who are not needed for the whole of the meeting, those with travel difficulties within South Africa and internationally, those with caring responsibilities and teaching commitments.

For both the Graduate Modelling Camp and the Study Group the software Zoom was used for all virtual meetings, both the main group and the breakaway study groups. An interactive Whiteboard was used for discussing mathematics. SharePoint online was used for depositing and sharing papers and other important documents. A copy of the MISG 2021 Technical

Information Guide, containing the Link to Zoom and all necessary information related to Zoom and SharePoint was sent to all participants before the meeting.

All participants had access to the necessary technology, either at home or at a University.

The Virtual MISG was officially opened on Monday morning by Professor Nithaya Chetty, Dean of Science, University of the Witwatersrand.

South African industry had been approached to submit problems during 2020. Four problems were submitted. On Monday morning each Industry Representative made a twenty-five minute virtual presentation in which the problem was described and outlined. The academics and graduate students then split into small study groups and worked in virtual break away rooms on the problem of their choice. Each problem was co-ordinated by an academic moderator and one or more student moderators. The role of the academic moderator was to co-ordinate the research on the problem during the week of the meeting and also to do preparatory work including literature searches before the meeting. The main function of the student moderators was to present short virtual presentations at the end of each working day reporting on the progress made that day. The moderators were in virtual contact with the Industry Representatives throughout the meeting. On Friday morning there was a full virtual report back session to industry. Each senior moderator, with assistance from the student moderators, made a twenty-five minute virtual presentation, summing up the progress made and the results that were obtained. Each Industry Representative then had five minutes to comment on the progress and the results that were reported. The MISG ended at lunch time on Friday.

The MISG was preceded by a Virtual Graduate Modelling Camp from Thursday 28 January to Saturday 30 January 2021. The objective of the Graduate Modelling Camp was to enable the graduate students to familiarize themselves with the Zoom technology and to provide them with the necessary background to make a positive contribution to the MISG the following week. The students were given hands-on experience at working collaboratively and interacting scientifically in small virtual study groups on problems of industrial origin and at making virtual presentations on their findings. Five problems were presented to the graduate students. The problems and presenters were:

• Designing a distributed personal-medical system	Jeff Sanders African Institute for Mathematical Sciences and University of Stellenbosch
• Covid-19 protection methods	Neville Fowkes University of Western Australia
• Turbulent two-dimensional jet	David Mason University of the Witwatersrand

- Predicting heart disease using logistic regression — Montaz Ali, University of the Witwatersrand
- Green roofs to mitigate the urban heat island — Gideon Fareo, University of the Witwatersrand

The graduate students worked in small virtual study groups on the problem of their choice.

The sponsors of the Virtual Graduate Modelling Camp and the Virtual MISG were:

- DST-NRF Centre of Excellence in Mathematical and Statistical Sciences
- African Institute for Mathematical Sciences

We thank the sponsors without whose support the Graduate Modelling Camp and the MISG could not have taken place.

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PROBLEM STATEMENTS

Problem 1. Value of domestic tourists in travel and tourism industry: Mpumalanga Province, South Africa

Industry: Tourism

Industry Representative: Precious Shabalala, University of Mpumalanga

Problem statement

The problem to be investigated is how customised packages of tourism products and services can be utilised to mitigate the challenge of catering for the domestic market. Presently tourism products and services are packaged in a manner that excludes the local market which is mainly poor and working class and favour the international market. Covid-19 has impacted negatively the entire travel and tourism industry. Mpumalanga tourism was mainly dependent on tourists from outside the province and international tourists. In order to attract domestic tourists, in particular from local areas, there is a need to comprehend the culture, accommodation, food and entertainment in the travel areas and then package it in an affordable manner. Socio-economic factors have an influence on the decision making of the local people because the majority fall below the poverty line and Mpumalanga is dominated by rural areas, The Study Group was asked to investigate customised packages aimed at providing value for money for domestic market tourists who have the potential to revive the tourist industry. In particular it was asked to develop a mathematical model to show how to attract, without running at a loss, local communities who are poor and working class, to venues (game reserves, heritage sites, panorama routes) and tourist facilities (hotels, camp sites and bed and breakfasts).

Problem 2. Developing a southern African tourism climate index

Industry: Tourism

Industry Representative: Jennifer Fitchett, School of Geography, Archaeology and Environmental Studies, University of the Witwatersrand

Problem statement:

In 1985 Mieczkowski developed the Tourism Climate index (TCI), a method to quantify, classify and compare the climatic suitability of various destinations for tourism and to determine changes in climatic suitability though time. This index was based on expert opinions regarding the climatic factors which are of importance to tourists and the relative weighting thereof. In the past five years a proliferation of new indices have emerged: the Climate Index for Tourism, the Beach Tourism Index, the Holiday Climate Index (Urban) and Holiday Climate Index (Beach) and Camping Climate Index. Each of these are argued to be more suitable than the Tourism Climate Index as they are based on the experiences of tourists, rather than expert opinion, yet each rely on only the experiences of tourists in the Northern Hemisphere.

This problem involves developing a Southern African Tourism Climate Index based on the experiences of tourists. These have been studied over the past 7 years through questionnaires, interviews and the coding of TripAdvisor reviews. These data sets will be available to the Study Group. The index will need to be appropriate to the tourist expectation of southern Africa (sun, sea and nature), be operational using the regularly collected meteorological variables in southern Africa and produce output measures of climatic suitability that are consistent with the experiences of tourists.

Problem 3. Masks and the spread of droplets and airborne virions

Industry: Medical

Industry Representative: Alex Welte, South African Centre for Epidemiological Modelling and Analysis, University of Stellenbosch

Problem Statement

Although quite a lot of work has been done to understand the effects of masks on the spread of infectious respiratory droplets and airborne pathogens, it is a complex subject and there are numerous questions to which there are no clear answers, or even solid heuristics. Currently, in the era of Covid 19, there is increased interest in the utility of masks to reduce the spread of infections.

This pitch proposes several avenues of investigation – suggesting that potential investigators choose something that appears amenable and interesting. First – further background:

There is considerable disagreement on the utility of different types of masks, although there are solid medical engineering type studies which indicate air and droplet flow under a variety of conditions, including normal breathing, coughing and sneezing, while wearing various barriers or none at all. Some argue that masks with ‘vents’ for exhaling, which are designed specifically to protect the wearer, offer little protection to others against possible pathogens exhaled by the wearer, while some argue that masks provide little protection either way. Clear shields are more comfortable to wear for some people and they certainly improve the quality of contact, most especially for people using lip reading as part of their communication strategy.

There is some evidence about the size of droplets which are both:

1. sufficiently large to be colonized by an infectious dose of virus or other pathogens (TB is spread by fully fledged bacterium and is MUCH larger than a virus): and

2. sufficiently small to remain airborne long enough to be inhaled by someone else passing through the space a few seconds or minutes later.

One possible investigation could be to explore regimes between no barrier and an impermeable barrier at some (tunable) distance from the face. One can consider the fate of droplets/particles of various sizes as they are exhaled in the presence of a tunable barrier (caught in/on the barrier, expelled through the barrier, expelled around the barrier). One can consider the difference between normal conversation and shouting or singing. It might be possible to provide specific guidance on practical topics like the importance of a good fit, the importance of the types of materials in masks, the utility of solid shields, the relative importance of masks for people who are breathing normally, versus people who are coughing or sneezing.

Problem 4. **Diffuser recycle rate**

Industry: Sugar cane processing

Industry Representative: Richard Loubser, Sugar Milling Research Institute, University of KwaZulu Natal, Durban

Problem Statement:

Sugar is extracted form shredded sugarcane using a counter current leaching process in a process unit called a diffuser. Shredded sugarcane with high sugar concentration enters the diffuser at one end and is drawn along the length of the diffuser, usually dragged by an arrangement of moving chains located under the bed of cane. Hot water, called imbibition, is poured onto the cane bed at the other end of the diffuser and percolates through the cane bed, leaching sucrose and other dissolved solids from the shredded cane.

It leaves the bed through a screen at the bottom and is collected in a tray. The juice is pumped forward to sprays above a point in the bed that is closer to the cane entry end, where it percolates through the bed and is collected in a tray adjacent to the first tray. The juice typically passes through 12 to 14 such stages, depending on the design of the diffuser. The concentration of sugar in the juice increases with each pass through the bed.

The objective of the diffuser design is to achieve flow that is as close to the ideal plug flow as possible. There are, however, limitations on how closely this can be achieved due to operational factors such as unsteady conditions and design factors that require a degree of recycle. Healthy cane will typically have 15% dissolved solids, 15% fibre (insoluble solids) and 70% water. Factories aim to add less than 300% imbibition on fibre (that is, 3 tonnes added water per tonne fibre in cane) to extract the sugar, such that the total water added is less than half the total mass of cane added. Shredded cane can absorb water so that its total water content is 85% of the total mass. A 300% imbibition rate, however, will only raise the water on cane to 79%. Therefore, without the total amount of water in the bed being increased through recycle, all the imbibition will be absorbed by the cane and little or no juice would be extracted without an additional mechanical squeezing step.

Mass transfer of sugar between the cane and the juice can only occur when there is good contact between the cane and the juice. Mass transfer is improved by increasing the liquid hold-up in the cane bed to increase the amount of cane surface in contact with liquid at any time. This is achieved by increasing the total amount of juice added to the top of the bed at each stage. Recycling of juice at a stage level is employed to increase the total juice flow per stage without increasing the total imbibition added to the diffuser. The more recycle used the greater the amount of juice in each stage.

However, recycle results in lateral dispersion of dissolved solids, thereby reducing the efficiency of counter-current extraction. Too much recycle may result in flooding of the cane bed, where the rate of juice addition exceeds the rate at which juice can percolate through the bed leading to pooling and lateral flow of juice on the top surface of the bed, with further negative consequences to the dissolved solids concentration profile. The operational object for the diffuser is to optimise the dissolved solids extraction through maintaining a sufficiently high percolation rate without compromising the dissolved solids concentration profile in the juice along the length of the diffuser. The flow rate of the juice in the bed needs to be maximised while minimising the degradation of the plug flow requirement.

The amount of recycle depends on several factors:

Controllable

- Height of the bed
- Horizontal bed speed
- Position of the sprays

Non-controllable

- Percolation rate (permeability)
- Length of the stage
- Amount of imbibition

A value for recycle of 30% is typically recommended but no scientific justification can be found for this value.

Questions to be explored:

1. What is optimum recycle fraction that should be used as a target for setting and controlling a diffuser?
2. Can a relationship between the controllable variables and non-controllable variables be derived that will enable the factory to achieve the optimum recycle?