UNSTEADY FLOW IN A DIFFUSER

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Abstract

The flushing of sugar from crushed and shredded cane fibres by gravity-facilitated flow takes place in a device called a *diffuser*. Crushed and shredded cane at the front end of the diffuser moves steadily toward the back end and fresh water is sprayed onto the top of the cane at the back end. The water is collected after percolating through the bed (removing sugar in the process) and is collected in trays from which it is pumped back and sprayed on the pulp again - this time further upstream. This counter-current extraction process is continued until the now concentrated sugar-water solution reaches the front end of the diffuser from where it is removed for further processessing. Operational difficulties arise because the permeability of the consignment can vary and this can cause either local flooding or drying, resulting in nonuniform (and therefore inefficient) sugar extraction. The project aim was to

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produce a model of the unsteady percolation process so that the effect of various measures to stabilise the process (such as varying feed speed or pump characteristics, or overall design) could be examined. The model produced is theoretically accurate and simple, understandable, and thus tailor-made for such investigations. Additionally issues associated with the start up of the diffuser were investigated.

1 Introduction

South African sugar refineries produce sugar for both the domestic and international markets and it is important to do this in the most efficient manner. One aspect of great importance is the washing of the shredded cane to extract the sugar. This takes place in a long mechanical structure called a *diffuser*. A long conveyor (approximately 60 m long) carries the fibre from where it is loaded to the other end of the diffuser. As it is carried along water is added near the outlet end, collected from where it flows out and then recycled further upstream. This is repeated until the now sugar-rich juice reaches the upstream end where it is collected for further processing. Along the length of the diffuser there are 12 bins or trays to collect the water/juice and 12 pumps that recycle the juice to an upstream inlet where it reenters the fibre bed via a spray. Cycling the juice in this way maintains a positive difference in concentration between the cane and the water, thus ensuring sugar is always being extracted by the water. It also has the effect that there is less water to boil off later in the process.

The level of water in the fibre is crucial at all times to maintain efficient operation. Ideally, every part of the shredded sugar (megasse) should have the juice pass over it once for each overhead spray, as this results in the maximum contact between the juice and fibre leading to greater transfer of the sugar to the water. However, inconsistencies in the properties of the fibre sometimes leads to a backing up of the liquid and consequent flooding of the surface, or a sudden draining of water, leaving a dry region.

Overflow of the juice onto the top of the substrate can lead to short-circuiting of the juice cycle or inefficient operation due to it flowing into the wrong catch-bin and being recycled incorrectly. Currently, the problem of flooding is treated by turning off the pump that is supplying that particular location, but this can lead to overfilling of one of the collection trays and flooding below the diffuser. Further exacerbating this problem is the extra flow from the pump once it is restarted, often leading to renewed flooding.

The process is started with a gradual filling of the diffuser with fibre, starting at a depth of around 0.5 m and gradually being raised to operating levels. The cane fibre travels most of the distance before the most downstream spray inflow is initiated with a goal of wetting as much as possible as quickly as possible. It is therefore of interest to model this process also.

So in summary the group was asked to consider the following problems of modelling some unsteady aspects of the process.

- Can we model the flow well enough to predict flooding or dry patch formation?
- If so, can we design a strategy to minimize problems should they arise or design a strategy to optimise the operation?
- Can we model the starting flow into dry fibre to ensure wetting of the megasse occurs as quickly as possible?

2 Modelling considerations

The percolating flow through the cane can be be modelled using a Darcy Law porous media flow model for fully saturated flow conditions, see [1, 2]. This well-established theory introduces a driving water potential ϕ given by

$$\phi = \frac{p}{\rho g} + y \tag{1}$$

where p is pressure, ρ is density, g is gravity and y is height above some reference level. The water flux $\mathbf{q}$ is given by the gradient of the potential as

$$\mathbf{q} = (u, v) = -k\nabla\phi \tag{2}$$

and $\mathbf{q} = (u, v)$ is the velocity vector (sometimes called specific discharge). The proportionality factor k is the permeability of the medium (in this case the shredded cane) and would need to be determined experimentally from the shredded cane matrix.

In saturated flow conditions, these equations are combined with conservation of mass $\nabla \cdot \mathbf{q} = 0$ so that,

$$\nabla \cdot \big(k\nabla\phi\big) = 0\ . \tag{3}$$

In many circumstances the permeability k is assumed to be constant (or close to it) and the result is that this equation reduces to Laplace's equation,

$$\nabla^2\phi = 0\ . \tag{4}$$

Under such saturated flow conditions the solution process is simple providing the air-water solution interface is known. However, if this interface needs to be determined as part of the solution process a difficult nonlinear problem results and approximate solutions are needed. In our case the location of this interface is central.

3 Optimal flow conditions for the diffuser

An ideal steady flow is depicted in Figure 1, see [3]. This solution is periodic, repeating over the "cells" beneath each spray. This example shows three such cells. The streamlines corresponding to a flux of $\mathbf{q} = \pm k/2$ run along the surface and divide one such cell from the next. The streamlines are bent at an angle that determines where the input from the spray will egress the bottom of the diffuser (and hence where the collection bin should be located).

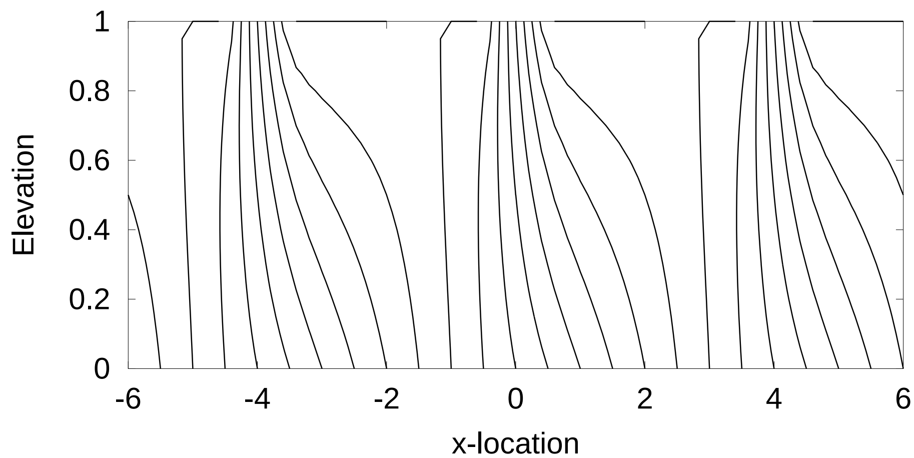

Figure 1: Steady, periodic Fourier series solution for the streamlines of the flow in a fully saturated section of the diffuser. The conveyor is moving left to right distorting the streamlines. See [3] for details.

Under the circumstances in this example, all of the cells will experience the same flushing history and the flushing rate will remain almost constant along the diffuser. Under uniform permeability conditions this ideal situation will be realised if the mass flux from the sprays matches the natural drainage rate, k.

If the flux from the sprays exceeds a flow equivalent to k then flooding will occur, while if the flux is less than this, some of the region near the surface will experience a reduction in flushing. Either way, nonuniform flushing will result.

Therefore, natural variations in k will lead to minor flooding or underflushing. In an operational context therefore, it is of interest to know how large these variations may be and how they will affect the outcome.

In a worst case scenario, if the flow rate is too low or there is a significant local increase in permeability then a plume may form in each section that does not meet up with plumes from the inlet sprays in the cells on either side. Figure 2 shows several such steady plumes for differing inflow rates. The details are given in [3]. This provides a lower bound on the necessary flow rates.

Therefore the ideal is a steady-state flow in which the juice passes from each spray into the substrate, is collected in the appropriate bin and then recycled upstream where the cycle repeats. These solutions indicate that the flow out through the bottom of the megasse will occur with velocity approaching $v = -k$, the permeability of the batch. This provides a guideline for the flow of fresh water at the most downstream spray - it should be approximately equal to $Q_{in} \approx kLW$ where L, W are the length and width of the catch-tray. It also provides the optimal conditions for the juice leaving a particular spray to enter the correct bin. The distance from the spray to the start of the desired catch-bin is equal to the conveyor speed multiplied by the time for the juice to pass vertically through the megasse, i.e. $L_{SD} \approx cH/k$. Therefore if the permeability k increases then c needs to be increased by the appropriate amount to maintain optimal collection.

A major problem is that it is very difficult to determine the flow conditions in the diffuser at any time as management is restricted to monitoring the water level at several narrow windows along the length of the diffuser and water levels in the catch trays beneath. Modelling can be used to infer conditions in the diffuser by determining some important parameters that can be obtained from this external monitoring.

4 Single Cell - estimating the plume

In order to better understand the flow in each "cell" of the diffuser we first consider the flow beneath a single spray jet. This should provide an estimate of the flow in a situation where the flow is not confounded by the horizontal movement of the conveyor. The equations are linear, so that the horizontal flow can be added later. However, here we wish to understand the way in which the liquid travels downward through the substrate.

One model for this is to regard the spray input as a slight overpressure on the surface, driving fluid into the megasse. The problem is nonlinear as the location of the air-water interface is unknown. However, we can estimate its

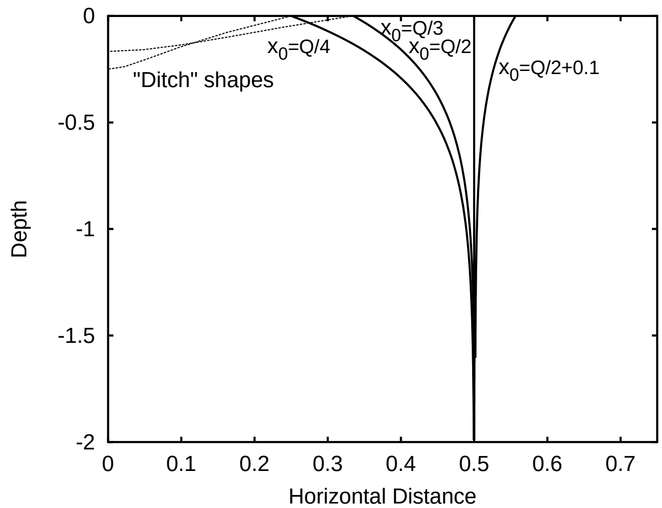

Figure 2: Approximate steady plume shapes for cases where the flow is insufficient to join up with neighbouring plumes. In higher flow cases the inflowing water actually spreads out across the surface before narrowing down as it flows down to its *terminal velocity* given by k. See [3] for details.

position from the solution of a problem in which the variation of the surface is small, thus linearizing the equations.

Suppose the "cell" has width $0 < \hat{x} < L_0$ and height $0 < \hat{y} < H_0$, and that the sprinkler is represented by an overpressure $\hat{\phi} = \hat{\phi}_0$ over some region $\hat{x}_1 < \hat{x} < \hat{x}_2$ on $\hat{y} = H_0$. We choose to scale the height with H_0, i.e. $\hat{y} = yH_0$, and the width with L_0, i.e. $\hat{x} = xL_0$, so that the domain is now $0 < x < 1, 0 < y < 1$, where x, y are nondimensional. This nondimensionalisation results in a modification to equation (4) to be in the form

$$\begin{aligned} \phi_{xx} \quad + \left(\frac{L_0}{H_0}\right)^2 \phi_{yy} = 0, & \qquad \text{subject to} \\ \phi_x(0,y) \;= 0, & \quad 0 < y < 1 \\ \phi_x(1,y) \;= 0, & \quad 0 < y < 1, \\ \phi(x,0) \;= 0, & \quad 0 < x < 1, \end{aligned} \tag{5}$$

and to introduce the overpressure along the top surface,

$$\phi(x,H) = \begin{cases} H & \text{if} \quad 0 < x < x_1, \\ 1 & \text{if} \quad x_1 < x < x_2, \\ H & \text{if} \quad x_2 < x < 1, \end{cases} \tag{6}$$

where $H < 1$. In this representation the diffuser is periodic so that conditions at the sides are that there is no flow from one cell to the next, i.e. $\phi_x = 0$ on

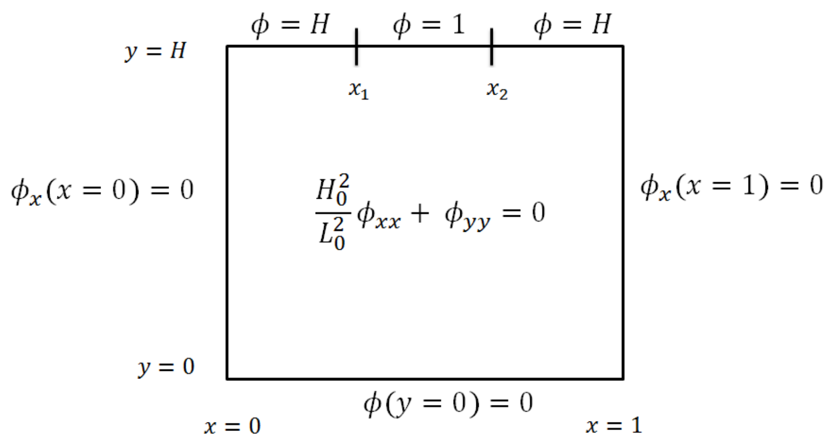

Figure 3: Schematic of non-dimensional single cell (stationary).

$x = 0, 1$, pressure is atmospheric at the bottom and downflow is driven by the pressure gradient.

The solution using separation of variables and Fourier series is

$$\phi = B_0 y + \sum_{n=1}^{\infty} B_n \cos\left(\frac{L_0}{H_0}\lambda_n x\right)\sinh(\lambda_n y), \quad \lambda_n = \frac{n\pi H_0}{L_0} \tag{7}$$

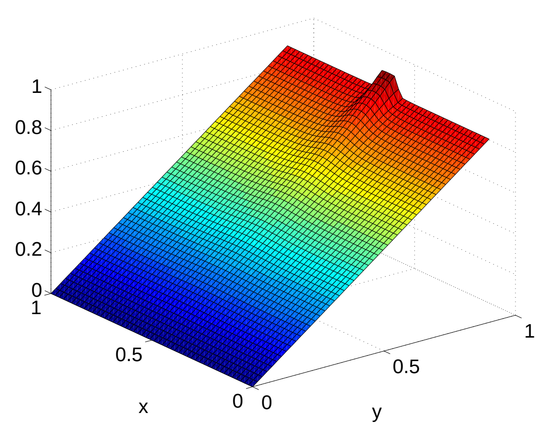

Figure 4: Surface plot of piezometric head ϕ within the cell.

with constants

$$B_0 = \frac{1}{H}\left[(H-1)(x_1-x_2)+H\right], \tag{8}$$

$$B_n = \frac{2}{\sinh(\lambda_n H)}\int_0^1 \phi(x,H)\cos\left(\frac{L_0}{H_0}\lambda_n x\right)dx\ . \tag{9}$$

The results for the potential (piezometric head) $\phi(x,y)$ are given in Figure 4 and it is clear that the graduation of ϕ is approximately linear from the bottom of the cell to the top, except very close to the inflow source.

Using this solution for ϕ we can estimate the shape of the air-water interface by integrating the kinematic condition along $y = H$:

$$\frac{H_0^2}{L_0^2}h_x = \frac{\phi_y}{\phi_x}, \qquad \text{with } h(x_2)=1, \quad \text{at } y = H\ . \tag{10}$$

Since the inflow rate is due to the difference between H and 1 ($H < 1$), a larger H means lower flow into the cell. Solutions for the free surface of the plume are given in Figure 5 for several different values of H.

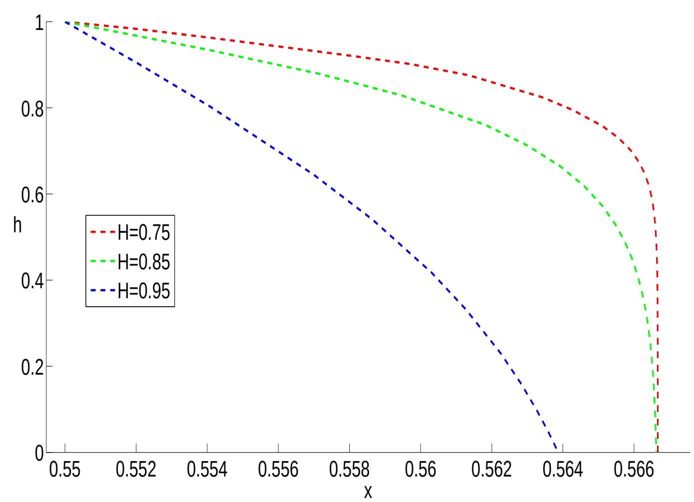

Figure 5: Solution for different H values.

These results therefore suggest that the gradient of ϕ is approximately linear except very close to the inflow point. In other words this means that the vertical flow velocity is approximately constant in ideal conditions, and in fact this velocity is close to $-k$, the permeability of the substrate, since the vertical component of velocity is $v = -k\phi_y \approx -k$ from (2) since $\phi_y \approx 1$. This agrees with the conclusions of the plume solution [3].

5 A simple box model

A simple model that begins to deliver some tangible results of the unsteady frame was developed. In this model each spray jet section was modelled as a single box as shown in Figure 6. A set of ordinary differential equations was derived for each box that take into account flow into the top and out through the bottom of the section. The boxes were assumed to be uniform throughout in permeability, although this could vary from box to box. A mass balance was derived for each. Water flowing out of the bottom was "collected" and placed in the section ready to be added upstream at the next inlet point. The inlets were assumed to occupy the full width of the box, so no account is taken of the horizontal shape of the plume.

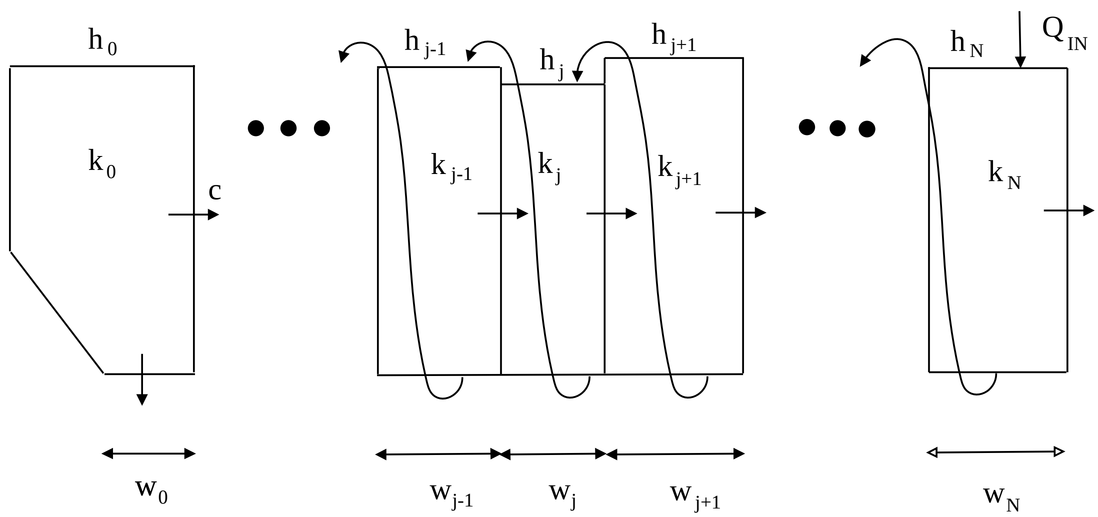

Figure 6: Schematic of 1-D Diffuser model.

The numerical scheme consists of:

$$\text{First column:} \qquad (w_0 + \beta h_0)\frac{\mathrm{d}h_0}{\mathrm{d}t} = -ch_0 - k_0 w_0 + k_1 w_1 \ ,$$

$$\text{Interior columns:} \qquad w_j \frac{\mathrm{d}h_j}{\mathrm{d}t} = w_{j+1}k_{j+1} - w_j k_j + \big(h_{j-1} - h_j\big)c \ , \qquad j = 1, 2, \ldots, N-1 \ ,$$

$$\text{Last column:} \qquad w_N \frac{\mathrm{d}h_N}{\mathrm{d}t} = -w_N k_N + \big(h_{N-1} - h_N\big)c + Q_{IN} \ ,$$

where h is liquid height, w is width of column, k is permeability, c is conveyor velocity and Q_{IN} is water flux in the last column.

A series of simulations was conducted using this model. Variations in permeability, conveyor speed, inflow speed and other control parameters can be investigated using this model. Although the model is not particularly sophisticated it does provide some general patterns of behaviour in different circumstances.

A local, short term increase in the permeability, k, progresses along the diffuser and causes some local flooding, but this passes relatively quickly through the system and the water level returns to the equilibrium level. Similarly a local decrease in k results in a local drying of substrate which gradually passes through the system.

A persistent change in permeability, k, appears to result in a persistent rise or fall in the water level in the diffuser. Thus in this situation, such as a new batch of pulp being introduced, the flow rate must be modified or the conveyor speed increased or decreased to accommodate the ongoing change in level. This simple model shows that a flooding event can be mitigated by adjusting the conveyor speed or adjusting the local flow rate.

The results of this simplistic approximation seem to indicate that the flow through the diffuser is relatively stable and robust. Local variations in flow or sugar pulp properties seem to quickly flow out of the system so that the equilibrium is resumed. More persistent modifications to the flow properties will quickly lead to a new equilibrium flow provided the flow rate is also appropriately modified.

However, in order to confirm these results a more accurate model is necessary. In the next section we describe a Lagrangian model that is a modification of the current model, but which allows a much more detailed resolution of the flow.

6 Lagrangian model

The box model described in Section 5 provides a simple method for investigating flooding events and local variation in permeability of the megasse, but does not provide the possibility to design strategies to mitigate the effects of these events. The resolution of the model is not sufficient to deal with items such as inflow spray width, and it does not provide a particularly accurate model of the location of the wetted region within the diffuser.

In order to answer these questions a more highly resolved model was developed in which a frame of reference moving with the megasse was adopted. In this model the megasse is represented by a set of narrow vertical columns in which the local water level is estimated. The physical properties (e.g. height, permeability) of each column remains the same as it moves along, but the water level in each varies due to inflow from the sprays, draining into the trays and exchange between adjacent columns.

6.1 Model design

The megasse is divided into N columns of width $w_j, j = 1, \ldots, N$, over the full length of the diffuser. Each column in the model has associated data $k_j, h_j, j = 1, \ldots, N$ representing the permeability of the megasse and the water level, respectively. The columns move along with the conveyor of the diffuser until they reach the end where they are removed from the system. At the most upstream end, new columns are added at each time to represent the new material being loaded. In order to simplify the computations and also to eliminate numerical diffusion, the time step is chosen so that the movement corresponds to exactly one column width. Thus, the properties in each cell can be updated after each time step by renumbering the index of the properties of each column.

Flow speed out of the bottom of the diffuser is approximately given by the permeability, $-k$, as shown by the plume solution in Section 4 and also in [3]. Water in the plume reaches close to this speed exponentially quickly once it enters the pulp, and so we can assume outflow will be close to this rate, so that outflow at the bottom from each column in time Δt is $Q_j^{out} \approx k_j w_j \Delta t, j = 1, 2, \ldots, N$. Inflow sprays produce a flux generated from the volume collected in bins further down. The columns into which they spray are computed as the columns move beneath them in each time step. The driving flow in the model is the flux from the most downstream spray inlet, since this is the water that is recycled throughout the system.

It is not difficult to show that the horizontal motion of the water between columns is an order of magnitude smaller than the vertical motion and the advection within the moving columns, but nonetheless this motion is computed using (2) after the spray input and bottom output and hence the new level in the column is computed at each time step. Finally, if the level of water in any column exceeds the maximum megasse height, then this liquid is allowed to spread and fill neighbouring columns that are not full. This replicates the process when flooding occurs.

Assuming an initial condition of constant water level in the diffuser, simulations show that a steady state quickly evolves (see Figure 7) from the upstream end as the conveyor proceeds, provided the inflow from the upstream end matches the through-flow. The flux of fluid through the diffuser is determined completely by the flow from the most downstream spray jet. To reach a steady flow condition, this flow should match the outflow through the bottom of the layer into the collection bin. This amount will be approximately equal to

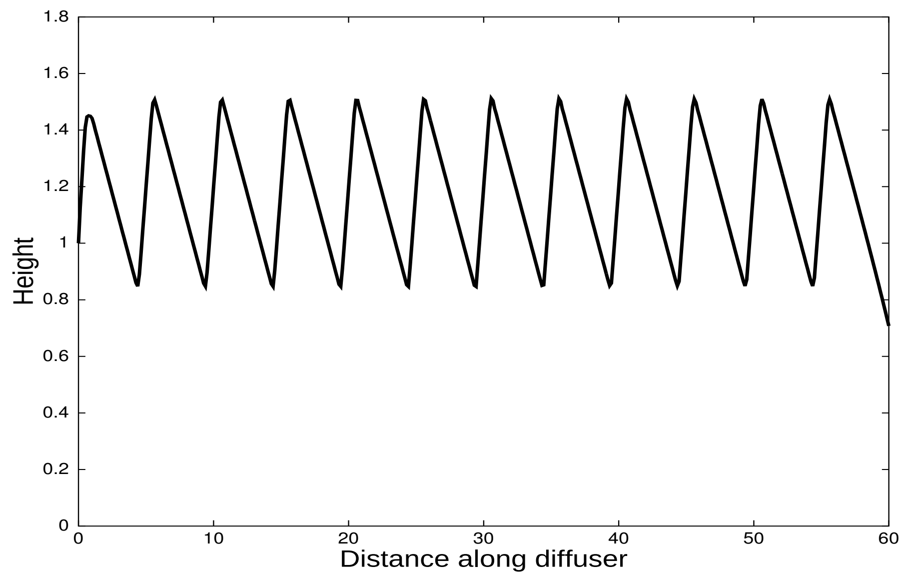

Figure 7: Steady state water level in the diffuser with $k = 0.008$, $c = 0.044$, spray width=0.5m. The slope of the draining surface is almost exactly given by $k/c \approx 0.18$.

the integral of the vertical velocity across the width of the bin, so $Q_{IN} = -kw$. This fluid is then recycled through the whole length of the diffuser.

A steady solution is shown in Figure 7 for the case with $k = 0.008$, the conveyor speed is $c = 0.044$ m/s and spray width=0.5m. In this configuration, the diffuser reaches a steady state after about 25 minutes. The peaks lie beneath the spray jets and as the conveyor proceeds the level simply drains downward, providing a sloping interface at an angle given approximately by $\tan\theta \approx k/c$, where k is permeability and c is the conveyor speed. The streamlines will generally be parallel to this part of the free surface. In ideal circumstances, if the flow is tuned to maximise water coverage and permeability remains constant, nothing more would need to be done.

However, the properties of the substrate are constantly changing as new sugar pulp is added to the conveyor. The consequences of this variation can be tested using the model and strategies developed to deal with it.

For example, consider a reduction in the width of the spray jets recycling the water. Figure 8 shows a comparison of steady state surface levels (over the first 25% of the diffuser) when a spray half-width of 0.25 m is used compared to one with a 0.5 m half-width. It is clear that there is very little difference between the surface levels.

A simulation was conducted to induce a small amount of flooding, i.e. where the level of the top of the substrate was set below the “natural” steady

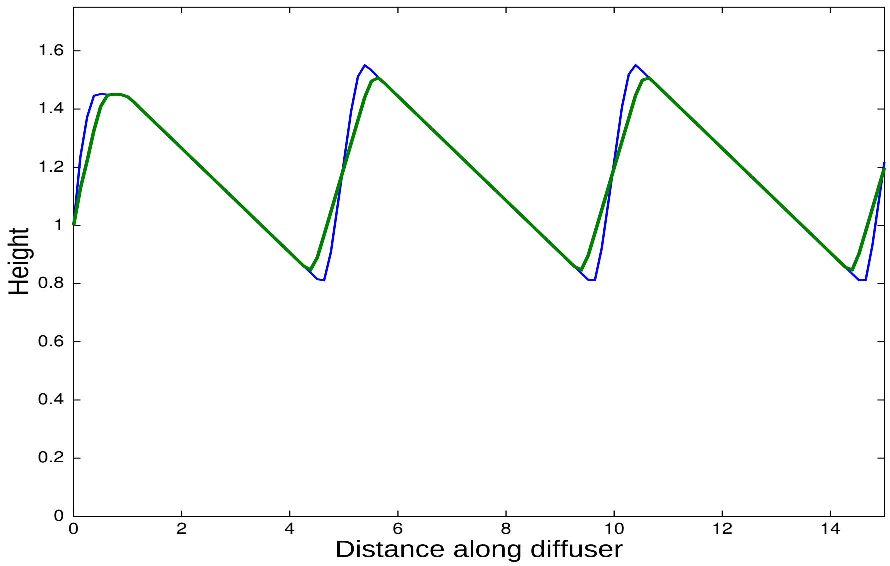

Figure 8: Steady state water level in the diffuser with $k = 0.008$, $c = 0.044$, spray half-width=0.5m (red) compared with spray half-width=0.25m (blue).

state level of the particular flow parameters. Figure 9 shows the results of a simulation for which the natural highest level is 1.55 m compared with one in which the megasse height is 1.4 m. The effect of the flooding is clearly evident as the excess water has spread across the surface and then soaked into the pulp. However, in a minor case such as this the end result differs very little from the case with a higher level. Liquid from this case will most likely still enter the correct bin, or only small amounts will flow into the wrong bin.

Further tests of an anomolous permeability being introduced to the diffuser for a short time can be seen in Figure 10 and Figure 11. The former shows the effect of a local decrease in permeability and the latter an increase. Slower draining can be seen to cause a higher level surface (Figure 10) at around the 30-35m mark along the diffuser, and also some flooding further down the diffuser as the water backs up. More rapid draining (Figure 11) causes a "dry" patch to form locally at about the same location (which also passes through the system), and slightly lower surface levels further down the diffuser as the water is pulled through more quickly. The results confirm the preliminary results of the simple box model, that a minor perturbation will eventually pass through the system. A more permanent change will require a modification to the inflow (or conveyor speed). However, simulations indicate that more extreme cases can lead to catastrophic changes in the efficiency with significant drying or flooding if appropriate adjustments are not made.

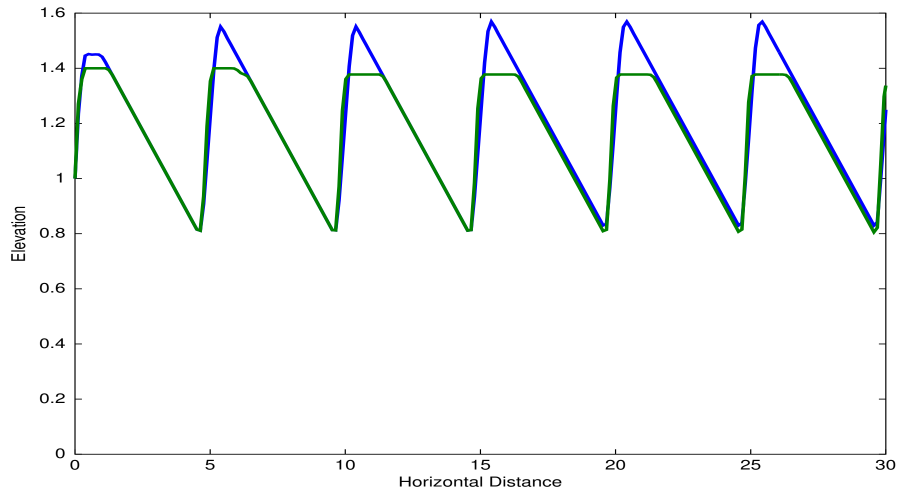

Figure 9: Steady state water level in the diffuser with $k = 0.008$, $c = 0.044$, spray half-width=0.25m and a substrate depth of 1.55m (blue) and 1.4m (red). The lower surface level results in flooding of the surface but this spreads and sinks into "dry" areas so that the overall steady solution is not much changed.

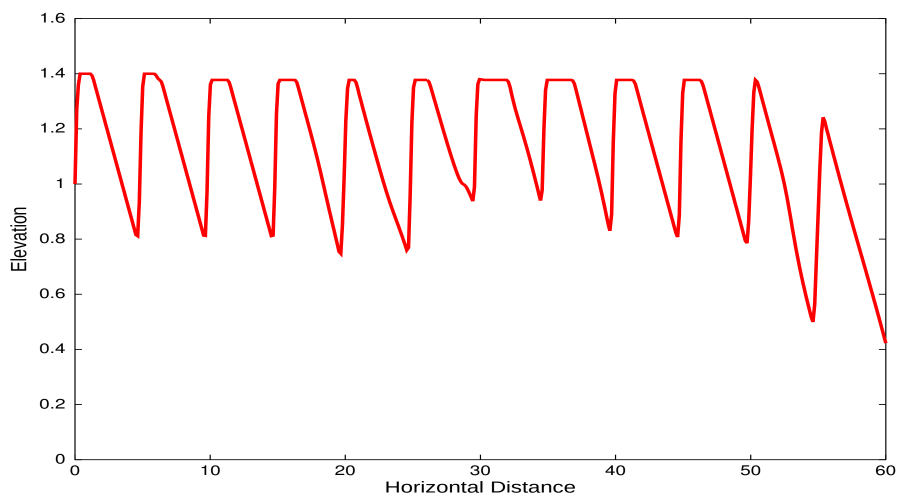

Figure 10: Water level in the diffuser with $k = 0.008$, $c = 0.044$, spray half-width=0.25m and a substrate depth of 1.4m, after 18 minutes. A 1.25m long "lump" with $k = 0.007$ was introduced after about 7 minutes. The level exhibits a slightly higher surface at the 30-35m mark, and a small amount of extra flooding further down as the water drains more slowly and hence backs up slightly. The lower levels at the end are due to the fact that a steady state has not yet formed completely.

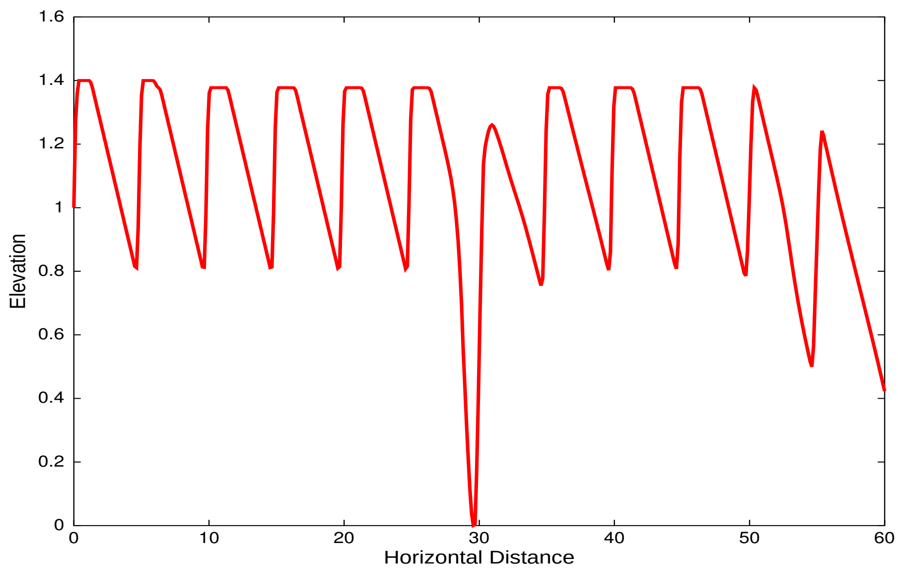

Figure 11: Water level in the diffuser with $k = 0.008$, $c = 0.044$, spray half-width=0.25m and a substrate depth of 1.4m, after 18 minutes. A 1.25m long "lump" with $k = 0.009$ was introduced after about 7 minutes. The more rapid draining has led to a complete emptying of the diffuser (and hence a dry patch) at the 30m mark. This dry patch, however, works its way through the system once the anomaly has passed and the steady state is resumed. The lower levels at the end are due to the fact that a steady state has not yet formed completely.

6.2 Comments

The Lagrangian model provides an effective tool for examining different scenarios for the flow within the diffuser. The results of a series of trials seem to indicate that the "steady" solution that evolves in ideal circumstances is quite robust to local changes in the conditions, resuming the steady state once the anomaly has passed. It also seems to show that minor flooding does not have a particularly dramatic effect on the outcome of the diffuser. Small amounts of flooding simply sink into the substrate either side and since they are only small amounts of fluid are unlikely to have a dramatic effect on the efficiency of the sugar extraction process.

Simulations reveal the strikingly periodic nature of the process. Very early, the water surface in the diffuser becomes periodic (even during the transient phase) and retains this periodicity throughout (except close to the two ends of the diffuser where end effects play a role). This is an important conclusion because it indicates that the process can be simulated very well by considering the behaviour of each individual cell using appropriate periodic boundary conditions.

The start-up simulations seem to indicate that there is some initial flooding even if the flow parameters are set correctly, but the steady solution is set up quite quickly and this would seem to deliver the optimal outcome.

This model is certainly preliminary but forms the basis for a more complete model in which full simulations could be conducted. The present version uses a very simplistic model for the behaviour of the pumps and is also limited to examining small changes to flow conditions rather than more dramatic changes. However, the simulations provide a strong indicator of behaviour and there is no reason why it could not be improved to provide an even better tool.

7 Flow initiation into the dry megasse

As a second problem a brief investigation was conducted into the flow as the diffuser was initiated. A simple model was formed based on a line source at the location of the first spray to be turned on. The source is assumed to force the water outward radially into the drier bed. Some of the front (approximately 10%) is absorbed in wetting the dry fibre, while the rest presses onward. We can iterate in time by simply recomputing the velocity due to the moving source at each time step. This source is moving relative to the bed and therefore has potential

$$\phi = -\frac{m}{2\pi} \log \left[(x - xs - ct)^2 + (y - ys)^2 \right]^{1/2}$$

where m is the source strength, (xs, ys) is the initial location of the source and ct gives the movement to the left relative to the bed. In other words the frame of reference is with the fibre and the source moves left with speed c. Once the moving interface hits either the boundary of the diffuser or the bottom, excess fluid is forced back upstream to a second source which is then initiated. The simple model requires more sophisticated treatment once the wetted regions from the two sources meet, but this is beyond the scope of the current investigation. As a "test-of-concept" this model appears to work quite well. To fully implement a scheme of this nature a more complete formulation including the pressure conditions on the moving front would need to be implemented, and also a model for the interaction of interfaces once they meet would need to be derived. However, some simple computations show promise in such a relatively simple scheme being able to approximately simulate the initial phase of the flow.

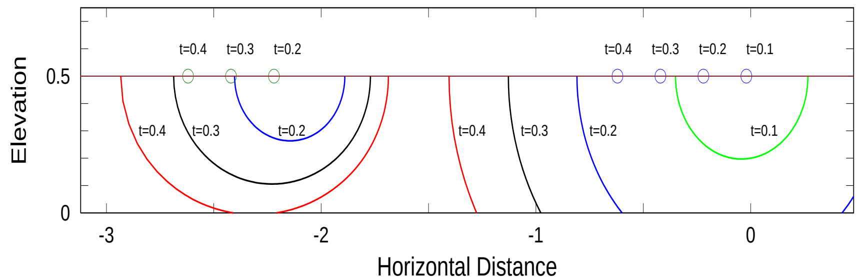

Figure 12: Results of a simulation using moving point sources to replicate the start-up. The circles indicate the sprinkler location at different times $t = 0.1, 0.2, 0.3, 0.4$, and the surface locations at different times are indicated. The flow from the second sprinkler doesn't start until after $t = 0.1$ when the flow from sprinkler 1 reaches the bottom of the diffuser.

8 Conclusions

The study group was asked to consider the flow in the process of washing sugar pulp as it travels along the diffuser. A model of a single plume to consider the dryness near the inlet sprays, a coarse compartment model to consider the flow with varying permeability and flows, a detailed Lagrangian model that more accurately resolves the flow levels and a model of the diffuser start up flow were developed.

The models were used to consider various aspects of the flow and the following conclusions were drawn.

- Vertical flow reaches the “terminal” velocity (k) exponentially quickly, so the plume width for any single spray jet can be estimated.

- The rate at which the collection bins fill can be used to estimate the local permeability (perhaps using pressure sensors and subtracting pumping rates). In theory it should be possible to estimate flow and permeability over the length of the diffuser from these data, thereby providing another guide to control the system.

- The flow conditions should, wherever possible be maintained at a flow rate and conveyor speed such that the distance from input sprinklers to collection bins is approximately cH/k where H is the depth of the sugar pulp.

- Minor variations in permeability that may lead to flooding or drying will usually return to the status quo quite quickly so might be best left alone, so flood events should be monitored for a short time to see if they dissipate.

- Flooding (or drying) events caused by a change in batch (so longer term change) will not clear and so a change in the flow rate throughout will be necessary.

- The steady state situation is almost a saw-tooth shape for the water level, with water levels rising under the spray and then draining between sprays. The steady state appears to be quite stable and will quickly adjust to a new steady state when parameters (permeability, inflow rates) change.

The Lagrangian model is a promising tool for further investigation. The model can be improved by improving the cycling of water through the system and by retaining more details of the characteristics within each cell. Coupling the model for sucrose derived in [3] to this one would allow simulation of the concentration throughout the system and thus a much more detailed optimization scheme, e.g. is the number of sprinklers and their separation optimal? Similarly, further work on the initiation model may provide an accurate representation of the early phase so that it too can be optimized.

References

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[3] C Breward, G Hocking, H Ockendon, C Please and D Schwendeman, Modelling the extraction of sugar from sugar cane in a diffuser, Proc. MISG South Africa 2012, University of the Witwatersrand, Johannesburg, pp 31–57.