PROCEEDINGS OF THE

MATHEMATICS IN INDUSTRY STUDY GROUP

2009

Mathematics in Industry Study Group South Africa
MISGSA 2009

The manuscripts for the Proceedings of the MISGSA were written by the problem moderators in consultation with the other members of the study group for that problem and the industry representative.

The Editor of the Proceedings was

Prof DP Mason	(University of the Witwatersrand)

The Editorial Board consisted of the members of the Scientific Committee:

The Proceedings consists of one Extended Summary and four Detailed Technical Reports.

The Extended Summary was not refereed.

The Detailed Technical Reports were submitted to the Editor. Each Report was refereed by two independent referees who were normally members of the Editorial Board and who were not also authors of the Report. Occasionally outside experts in the field were used as referees. On the recommendation of the referees the four Reports were accepted for the Proceedings subject to corrections and minor revisions. The Editor would like to thank the outside experts for their assistance in refereeing the Reports for the Proceedings.

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PREFACE

The sixth Mathematics in Industry Study Group (MISG) Workshop in South Africa was held in the School of Computational and Applied Mathematics at the University of the Witwatersrand, Johannesburg, from Monday 26 January to Friday 30 January 2009.

There were sixty-one participants at the MISG. Nine academic staff, thirty-eight postgraduate students, eight Industry Representatives and six invited overseas guests attended. The invited guests were:

Dr Hilary Ockendon:	OCIAM, University of Oxford, England
Dr Stephen Peppin:	OCCAM, University of Oxford, England
Dr Jonathan Whitely:	OCCAM, University of Oxford, England
Prof Steven Damelin:	Georgia Southern University, United States of America
Dr Jean Charpin:	University of Limerick, Ireland
Dr Antony Owinoh:	Free University Berlin, Germany

The South African Universities which were represented were:

Cape Peninsula University of Technology
University of Johannesburg
University of KwaZulu-Natal
North-West University
University of Pretoria
University of Stellenbosch
Vaal University of Technology
University of the Witwatersrand

Two universities from the rest of Africa sent participants:

Kigali Institute of Science and Technology, Rwanda
University of Malawi

The CSIR Pretoria was represented for the first time.

The MISG Workshop was opened by Professor Andrew Crouch, the Dean of the Faculty of Science at the University of the Witwatersrand.

The MISG Workshop followed the established format for MISG meetings held in the United Kingdom, Australia, New Zealand, Canada, Asia and the United States. South African industry had been approached to submit problems during the second half of 2008. Six problems were submitted. On Monday morning each Industry Representative made a twenty-five minute presentation in which he described the problem and outlined what he thought needed to be done. On Tuesday, Wednesday and Thursday the academics, together with the graduate students, worked in small study groups on problems which suited their interest and expertise. Each problem was co-ordinated by a senior moderator and a student moderator. The role of the senior moderator was to co-ordinate the research on the problem during the week of the meeting and also to do preparatory work including literature searches before the meeting. The main function of the student moderator was to present the fifteen minute progress report on Wednesday afternoon. The moderators were in contact with the Industry Representative on Tuesday, Wednesday and Thursday. On Friday morning there was a full report back session to industry. Each senior moderator made a twenty-five minute presentation, summing up the progress made and the results that were obtained. Each Industry Representative then had five minutes in which to make comments on the progress and results which were reported. The MISG ended at lunch time on Friday.

Two short presentations were made on Tuesday and Wednesday after morning tea:

Stephen Peppin	"*Oxford Centre for Collaborative Applied Mathematics*"
Colin Wright	"*Centre for High Performance Computing, Cape Town*"

The main contribution made during the week of the MISG was to expose the industrial problems to the mathematics community and to do modelling and simulations. Work continued on the problems after the meeting ended. In March 2009 an equation-free Executive Summary, not more than two pages in length, for each problem was given to each Industry Representative. The Executive Summary was designed to inform Management of the progress made at the MISG on their problem. In the Proceedings of the MISG the mathematical progress made on each problem up to July 2009 is presented and suggestions for further work are made. Moderators with the most active members of their group and the Industry Representative will be encouraged to publish their results in international journals.

A MISG brings together mathematicians to work on and solve research problems of industrial origin. Mathematical solutions will assist South African industry to become more efficient and competitive thereby creating jobs and contributing to the prosperity of South Africa. Mathematicians in turn see the challenges facing industry. By working in small groups with experienced industrial mathematicians academics receive training in solving problems from industry. New collaborations are established within South Africa and also internationally with the invited guests. Higher degree students are encouraged to participate in the small study groups and the work done could develop into suitable mathematics in industry topics for Masters dissertations and PhD theses. By demonstrating to companies that mathematics can be used successfully to solve problems in industry, job opportunities will be created in industry for graduates in the mathematical sciences. Applied industrial problems can also lead to problems in basic research. Some of the problems should provide innovative teaching material since mathematical modelling plays a central role in the solution process.

The MISG was proceeded on Saturday 24 January by a Mathematics in Industry Graduate Student Workshop. Nineteen graduate students participated in the Workshop. The aim of the workshop was to prepare the graduate students for the MISG which started the following Monday. The Facilitator of the Workshop was Dr Jean Charpin of the University of Limerick, Ireland. He was assisted by Professor David Mason of the University of the Witwatersrand. The Workshop was officially opened by Professor Yunus Ballin, Deputy Vice-Chancellor (Academic) of the University of the Witwatersrand. Four problems were presented to the graduate students:

Cooling of concrete
Transport of mining waste material
Effect of a red traffic light or an accident on traffic flow
Wrinkling of paper labels on bottles

The graduate students then split into study groups and worked on the problem of their choice throughout the day. Each group then presented their results at a report back session on Saturday evening.

The sponsors of the MISG were:

National Research Foundation (NRF), Pretoria, South Africa
Dean's Discretionary Fund, Faculty of Science Research Committee, University of the Witwatersrand
Department of Education, South African Government

We thank the sponsors without whom the Mathematics in Industry Study Group meeting could not have taken place.

LIST OF DELEGATES

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Motlhabane, Lebogang – Gauteng Department of Community Safety.

Musingwini, Cuthbert − School of Mining Engineering, University of the Witwatersrand, Johannesburg.

Raol, Jitendra – Mobile Intelligent Autonomous Systems Group, CSIR, Pretoria.

Sears, Michael – School of Computer Science, University of the Witwatersrand, Johannesburg.

Ubbink, Onno – Pebble Bed Modular Reactor (Pty) Ltd. Johannesburg.

Utete, Simukai – Mobile Intelligent Autonomous Systems Group, CSIR, Pretoria.

Wright, Colin – Centre for High Performance Computing, Cape Town.

PROBLEMS

For each problem submitted by industry, the title of the problem, the industry presenting the problem, the industry representatives and the academic moderators are listed below.

Problem 1.

Title: Strategic placement of road blocks in the fight against crime in a South African city
Industry: Gauteng Department of Community Safety
Industry Representative: Lebogang Motlhabane
Moderators: David Sherwell and David Mason
Student Moderators: Dario Fanucchi, Tanya La Foy, Terry Oliphant

Problem 2.

Title: A linear programming and stochastic analysis of mining replacement rate for typical bushveld complex platinum reef conventional mining under variable geological losses
Industry: Platinum mining
Industry Representative: Cuthbert Musingwini and Richard Minnitt
Moderator: Montaz Ali
Student Moderator: Thato Dikgale

Problem 3.

Title: An overview and ideas on autonomous robot path planning algorithms
Industry: Mobile Intelligent Autonomous Systems Group, CSIR, Pretoria
Industry Representatives: Simukai Utete and Jitendra Raol
Moderator: Mapundi Banda
Student Moderator: Dario Fanucchi

Problem 4.

Title: Pebble bed: reflector treatment and pressure velocity coupling
Industry: Pebble Bed Modular Reactor
Industry Representative: Onno Ubbink
Moderators: Jean Charpin
Student Moderator: Jean Medard Tchoukouegno Ngnotchouye

Problem 5.

Title: Merging of image data from multiple sensors
Industry: Anglo American Corporation
Industry Representative: Michael Sears
Moderator: Steven Damelin
Student Moderator: Kerry-Anne Cawse

Executive Summaries

A brief description of each problem is given followed by the equation-free Executive Summary for the problem

STRATEGIC PLACEMENT OF ROAD BLOCKS IN THE FIGHT AGAINST CRIME IN A SOUTH AFRICAN CITY

Gauteng Department of Community Safety

Industry Representative
Lebogang Motlhabane, Gauteng Department of Community Safety, Johannesburg

Moderators
David Sherwell and David Mason, School of Computational and Applied Mathematics, University of the Witwatersrand, Johannesburg.

Student Moderators
Dario Fanucchi, Tanya La Foy and Terry Oliphant, School of Computational and Applied Mathematics, University of the Witwatersrand, Johannesburg.

Description
It has proved effective in the fight against crime to concentrate on the search of motor cars. The structure of a road block is important as is the strategic location of the road block. Certain cities have known target areas and known hiding places within a police precinct. We imagine that a crime is immediately reported, that the location of the crime is known and that the perpetrators flee the scene of the crime by motor car. The precinct is patrolled by roaming police cars which are in radio contact with police Headquarters. This is the status in the Honeydew precinct of Gauteng. The Study Group was asked to devise a strategy to establish road blocks to intercept criminals. A road block disrupts the traffic flow and can produce long tailbacks of vehicles. The Study Group was also asked to investigate the effect of the road block on the traffic flow and the time taken for the effects of the road block to clear after it has been removed.

Executive summary

The map of Honeydew is a graph on which the scene of the crime and the likely hiding places of the criminals are plotted. The max-flow algorithm of FORD-FULKERSON and the approximate minimum cuts algorithm of BENZCUR-KARGER were used to find the smallest number of cuts of the graph that will block passage from the scene of the crime to the hiding place. It is shown that on the map of Honeydew three road blocks are usually adequate to trap the criminals, if patrol cars can be dispatched quickly enough. There are usually about five patrol cars in Honeydew and so there is safety back-up. The Study

Group suggests that the Honeydew police can be efficiently supported by appropriate mathematical software which can be implemented in a user-friendly way.

A model in which the traffic velocity is a linear function of the traffic density was used to investigate the effect of the road block on the traffic flow. The road consists of two lanes in one direction. One lane is closed for a short distance by the road block. The length of the tailback at the entrance to the road block and the traffic flux at the exit to the road block are calculated. The road block is removed after a time T. It is found that the maximum length that the tailback grows is a linear function of T. The effect of the road block on the traffic flow depends on three parameters, the density of the oncoming traffic, the time T and the ratio λ of the speed limit in the road block to the speed limit on the open road. The police at the road block can manage the congestion caused by the road block by adjusting T and λ. We suggest that police/community relations can be improved by careful management of road blocks.

A LINEAR PROGRAMMING AND STOCHASTIC ANALYSIS OF MINING REPLACEMENT RATE FOR TYPICAL BUSHVELD COMPLEX PLATINUM REEF CONVENTIONAL MINING UNDER VARIABLE GEOLOGICAL LOSSES

Platinum Mining

Industry Representatives
Cuthbert Musingwini and Richard Minnitt, School of Mining Engineering, University of the Witwatersrand, Johannesburg.

Moderator
Montaz Ali, School of Computational and Applied Mathematics, University of the Witwatersrand, Johannesburg.

Student Moderator
Thato Dikgale, School of Computational and Applied Mathematics, University of the Witwatersrand, Johannesburg.

Description
The Bushveld Complex in South Africa is currently the largest source of known platinum reserves in the world. Conventional mining is the most prevalent method of mining the platinum. In conventional mining stopes are generated to give access to the platinum. Mining replacement rate is the rate at which development generates new stopes to replace depleting ones thus sustaining production. It is often necessary to defer development as far as possible into the future because it is a cost. However, deferring development is a drawback to operational flexibility and inadequate flexibility leads to failure in meeting planned production targets or operating inadequately prepared working areas that compromise safety. The problem is further compounded by geological losses in the form of potholes, dykes and faults whose exact location, extent and nature are not known with certainty prior to mining. These losses are generally encountered in a stochastic way.

The Study Group was asked to optimise development and stoping rates by adopting an appropriate mining replacement rate taking into account the stochasticity of losses.

Executive summary

Existing operations use mining replacement rates based on empirical approaches. During the meeting the Study Group was able to develop a mathematical model. The mathematical model became a stochastic linear programming problem which was then solved using existing routines with some ad hoc considerations. Mining replacement rates within the range of 10% to 60% for geological losses, which are typical of Bushveld Complex platinum reefs, were investigated.

AN OVERVIEW AND IDEAS ON AUTONOMOUS ROBOT PATH PLANNING ALGORITHMS

Mobile Intelligent Autonomous Systems Group, CSIR

Industry Representative
Simukai Utete and Jitendra Raol

Moderator
Mapundi Banda, School of Computational and Applied Mathematics, University of the Witwatersrand, Johannesburg

Description

The objective is to identify appropriate approaches for autonomous robot motion. The challenge is to identify a path for a robot to take from its initial or starting position to a goal position. The terrain in which the robot navigates will be unstructured. In addition there will also be obstacles present in the terrain. The obstacles will be static as well as dynamic or mobile. The proposed approaches will be required to define a navigation trajectory for the robot in addition to proposing an obstacle avoidance strategy.

Executive summary

Depending on the accuracy and the efficiency four possible approaches have been identified from literature. The following are possible ideas for defining a navigation path for the robot:

a) Potential Field/Level Set Method: A potential field will be designed to direct the robot from its initial position to its goal. The potential field will be defined in such a way that obstacles repel the robot and the target/goal attracts the robot to itself. The idea is that using such a field the robot can identify a path that avoids the 'repelling' field and directs the robot to the 'attractive' goal.

b) Calculus of Variation/Optimal Control: In this approach the robot motion planning will be formulated as a calculus of variation or an optimal control problem. The robot will minimize a certain cost subject to some constraints which describe the trajectory which it maps out as it navigates its path to the goal.

c) Network Algorithm: In this case a network algorithm, the *A** algorithm, will be used to navigate the terrain. The terrain with obstacles will be represented as a network and the initial and target positions of the robot identified. To define the network a random sample of navigable points will be connected in succession until a collection of possible paths from the initial to the target position have been mapped out. The *A** algorithm will thereafter be used to navigate the network.

Initially, static obstacles will be considered. As an extension to the approaches dynamic obstacles will be included.

PEBBLE BED: REFLECTOR TREATMENT AND PRESSURE VELOCITY COUPLING

Pebble Bed Modular Reactor

Industry Representative
Onno Ubbink, Pebble Bed Modular Reactor (Pty) Ltd.

Moderator
Jean Charpin, Department of Mathematics and Statistics, University of Limerick, Ireland

Description
PBMR is a gas cooled pebble bed nuclear reactor of approximate cylindrical shape. The reactor core is filled with around 450 000 fuel pebbles containing low enriched uranium, located in an annular region close to the graphite centre of the reactor. The fuel is consumed progressively and this nuclear reaction releases heat. Helium flows through the hot pebbles to cool down the system and the heat transferred to the gas is converted into electricity through a turbine.

PBMR Ltd aims to simulate the helium flow and temperature in the pebbles speedily and accurately. This is currently achieved by calculating the values of the required parameters for a very limited number of points. They asked the group to investigate numerical difficulties they have encountered with the method they currently use. PBMR Ltd are looking for solutions that would ideally allow them to keep their present model and would only change their numerical method marginally.

Executive summary
The study group first investigated the flow model in the pebbles. The model used by PBMR Ltd is based on the Navier-Stokes equations where the flow resistance is modelled using a turbulent friction law. We briefly considered an alternative approach based on Darcy's law but decided the present model was appropriate in the given parameter regime.

The group then concentrated on numerical aspects:

- Flow calculation. Changes in the flow resistance are large and discontinuous when the gas reaches the pebbles. This causes unrealistic oscillations in the numerical solution around the gas-pebble interface. A method was found in the literature to overcome this problem without resorting to major recoding [1].

- Temperature. PBMR Ltd uses two types of meshes in their simulations: cell centred and vertex centred. In theory, the results should be independent of the mesh but in practice, PBMR Ltd observes differences up to thirty degrees in the neighbourhood of the interface between the different layers of the reactor. The group developed a numerical method based on integration and local analytical solutions to produce equivalent and accurate results with the two types of mesh and showed that this would work in a simple one-dimensional case.

Reference

J.Mencinger and I. Zun. On the finite volume discretization of discontinuous body force field on collocated grid: Application of VOF method, *J. of Computational Phys*, **221**, (2007), 524-538.

MERGING OF IMAGE DATA FROM MULTIPLE SENSORS

Anglo American Corporation

Industry Representative
Michael Sears, School of Computer Science, University of the Witwatersrand, Private Bag 3, Wits 2050, Johannesburg

Moderator
Steven Damelin, Department of Mathematical Sciences, Georgia Southern University, United States of America

Description
In remote sensing an important and persistent problem is the comparison of images of the same scene taken by different sensors or even different spectrometers of the same sensor. The problem is that the geography of the scene as well as the positions of the sensors make it extremely difficult to match the pixels in the two images using a model based approach, that is, calculating where each pixel should be on the basis of accurate (but not exact) information on the positions of the sensors and the topography of the scene.

Executive summary
The group investigated this issue initially in general terms. However, it soon became clear that the problem was too diverse and challenging in that context; just the literature review would take more than the week available. We therefore focussed on a particular problem which had a variety of aspects of the general problem and thus could provide insights into the general problem, yet also had additional specific information that would be available to provide a method of attack.

We considered the case of the Hyperspectral Core Imager owned by Anglo Gold Ashanti. The instrument generates hyperspectral data of split drill core – which enabled the identification of minerals present - but also takes high resolution digital images in natural colour using a separate digital frame-grab camera. The problem of matching the hyperspectral data to the much higher spatial resolution camera data is complicated by a number of factors not least that the camera takes its pictures sometime after the hyperspectral data is captured.

A number of different approaches were considered by the group. The most innovative was the idea of using the multiple hyperspectral “images”, that is, the images at each wavelength recorded, as multiple pictures of the data, in a similar way to the approach of looking at temporal change detection by looking at multiple images taken at different times. This approach will be considered further in project work during the year.