CONTENTS

PREFACE

The Fourth Mathematics in Industry Study Group (MISG) Workshop in South Africa was held in the School of Computational and Applied Mathematics at the University of the Witwatersrand, Johannesburg, from Monday 29 January to Friday 2 February 2007.

There were sixty participants at the MISG. Twenty University staff, one Postdoctoral Fellow, twenty-eight postgraduate students, eight Industry Representatives and three invited overseas guests attended. The guests were:

Professor Verdiana Masanja:	Kigali Institute of Science and Technology Rwanda
Professor Alejandro Chacur:	University of Concepcion Chile
Dr Jean Charpin:	University of Limerick Ireland

The South African Universities which were represented were:

University of Cape Town
University of Johannesburg
University of Kwa Zulu-Natal
University of Limpopo
North West University
University of Pretoria
Vaal University of Technology
University of Venda
University of Western Cape
University of the Witwatersrand

The MISG Workshop was opened by Professor Ramesh Bharuthram, the Dean of the Faculty of Science at the University of the Witwatersrand.

The MISG Workshop followed the established format for MISG meetings held in the United Kingdom, Australia, New Zealand, Canada, Asia and the United States. South African industry had been approached to submit problems during the second half of 2006. Eight problems were submitted. Five of the problems were investigated at the MISG and the choice was determined by the interests and experience of the participants. On Monday each Industry Representative made a twenty-five minute presentation in which he described the problem and outlined what he thought needed to be done. On Tuesday, Wednesday and Thursday the academics worked in small groups on problems which suited their interest and expertise. Each problem was managed by a moderator whose role was to co-ordinate the research on the problem during the week of the meeting and also to do preparatory work including literature searches before the MISG meeting. Four of the five moderators were from South Africa. Each moderator was in contact with the Industry Representative on Tuesday, Wednesday and Thursday. On Wednesday afternoon a graduate student from each study group presented a ten-minute progress report on the problem. On Friday morning there was a full report back session to industry. Each moderator made a twenty-five minute presentation, summing up the progress made and the results that were obtained. Each Industry Representative then had five minutes in which to make comments on the progress and results which were reported. The MISG ended at lunch time on Friday.

An invited lecture was given after lunch on Tuesday and Wednesday by applied mathematicians with experience at solving problems from industry. The aim of the lectures was to show how mathematics could be used to solve problems in industry:

Prof Verdiana Masanje	"*Swash zone flow and applied mathematics at the University of Dar es Salaam and Kigali Institute of Science and Technology*"
Prof Alejandro Chacur	"*Valuing methods and application in the pharmaceutical industry*"

The main contribution made during the week of the MISG was to expose the industrial problems to the mathematics community and to do modelling and simulations. Work continued on the problems after the meeting ended. In March 2007 an equation-free Executive Summary, not more than two pages in length, for each problem was given to each Industry Representative. The Executive Summary was designed to inform Management of the progress made at the MISG on their problem. In the Proceedings of the MISG the mathematical progress made on each problem up to December 2007 is presented and suggestions for further work is made. Moderators with the most active members of their group and the Industry Representative will be encouraged to publish their results in international journals.

A MISG brings together mathematicians to work on and solve research problems of industrial origin. Mathematical solutions will assist South African industry to become more efficient and competitive thereby creating jobs and contributing to the prosperity of South Africa. Mathematicians in turn see the challenges facing industry. By working in small groups with experienced industrial mathematicians academics receive training in solving problems from industry. New collaborations are established within South Africa and also internationally with the invited guests. Higher degree students are encouraged to participate in the small study groups and the work done could develop into suitable mathematics in industry topics for Masters dissertations and PhD theses. By demonstrating to companies that mathematics can be used successfully to solve problems in industry, job opportunities will be created in industry for graduates in the mathematical sciences. Applied industrial problems can also lead to problems in basic research. Some of the problems should provide innovative teaching material since mathematical modelling plays a central role in the solution process.

The sponsors of the MISG were:

National Research Foundation (NFR), Pretoria, South Africa
Anglo American Chairman's Fund
Dean's Discretionary Fund, Faculty of Science, University of the Witwatersrand

We thank the sponsors without whom the Mathematics in Industry Study Group meeting could not have taken place.

LIST OF DELEGATES

Academic

Ali, Montaz – School of Computational and Applied Mathematics, University of the Witwatersrand, Private Bag 3, Wits 2050, Johannesburg.

Banda, Mapundi – School of Mathematical Sciences, University of Kwa Zulu-Natal, Private Bag X54001, Durban 4000.

Bassett, Bruce – Department of Mathematics and Applied Mathematics, University of Cape Town, Private Bag, Rondebosch 7701.

Benyah, Francis – Department of Mathematics, University of the Western Cape, Private Bag X17, Bellville 7535.

Chacur, Alejandro – Department of Industrial Engineering, University of Concepcion, Concepcion, Chile.

Charpin, Jean – Department of Mathematics and Statistics, University of Limerick, Limerick, Ireland.

Khalique, Masood – Department of Mathematical Sciences, North West University, Mafikeng Campus, Private Bag X2046, Mmabatho 2735.

Lombe, Mubanga – Department of Mathematics and Statistics, University of Zambia, Lusaka, Zambia.

Makinde, Oluwole – Department of Applied Mathematics, University of Limpopo, Private Bag X1106, Sovenga 0727.

Masanja, Verdiana – Kigali Institute of Science and Technology, Rwanda.

Masinga, Londiwe – School of Computational and Applied Mathematics, University of the Witwatersrand, Private Bag 3, Wits 2050, Johannesburg.

Mason, David – School of Computational and Applied Mathematics, University of the Witwatersrand, Private Bag 3, Wits 2050, Johannesburg.

Moitsheki, Joel – Department of Mathematics, Vaal University of Technology, Private Bag X021, Vanderbijlpark 1900.

Moyo, Simiso – Department of Mathematics and Applied Mathematics, University of Venda.

Mureithi, Eunice – Department of Mathematics and Applied Mathematics, University of Pretoria, Private Bag X650, Pretoria 0001.

Myburgh, Colin – School of Computational and Applied Mathematics, University of the Witwatersrand, Private Bag 3, Wits 2050, Johannesburg.

Myers, Tim – Department of Mathematics and Applied Mathematics, University of Cape Town, Private Bag, Rondebosch 7701.

Napier, John – School of Computational and Applied Mathematics, University of the Witwatersrand, Private Bag 3, Wits 2050, Johannesburg.

Neygebauer, Igor – Department of Applied Mathematics, National University of Rwanda, Rwanda.

Phiri, Patrick – Department of Mathematical Sciences, North West University, Mafikeng Campus, Private Bag X2046, Mmabatho 2735.

Ramalingam, Ravindran – School of Computational and Applied Mathematics, University of the Witwatersrand, Private Bag 3, Wits 2050, Johannesburg

Sjoberg, Astri – Department of Applied Mathematics, University of Johannesburg, P.O. Box 524, Auckland Park 2006, Johannesburg.

Sizani, Bandile – School of Economic and Business Science, University of Kwa Zulu- Natal, Private Bag X52001, Durban 4000.

Postdoctoral Fellows

Mitchell, Sarah – Department of Mathematics and Applied Mathematics, University of Cape Town, Private Bag, Rondebosch 7701.

Students

Andrianjafinandrasana, Naval – School of Computational and Applied Mathematics, University of the Witwatersrand, Private Bag 3, Wits 2050, Johannesburg.

Anthonyrajah, Marlyn – School of Computational and Applied Mathematics, University of the Witwatersrand, Private Bag 3, Wits 2050, Johannesburg.

Dwane, Sithembinkosi – Department of Applied Mathematics, University of Limpopo, Private Bag X1106, Sovenga 0727.

Fareo, Gideon – School of Computational and Applied Mathematics, University of the Witwatersrand, Private Bag 3, Wits 2050, Johannesburg.

Fredericks, Ebrahim – School of Computational and Applied Mathematics, University of the Witwatersrand, Private Bag 3, Wits 2050, Johannesburg.

Gabere, Musa – School of Computational and Applied Mathematics, University of the Witwatersrand, Private Bag 3, Wits 2050, Johannesburg.

Hlozek, Renée – Department of Mathematics and Applied Mathematics, University of Cape Town, Private Bag, Rondebosch 7701.

Jacquier, Robert – School of Computational and Applied Mathematics, University of the Witwatersrand, Private Bag 3, Wits 2050, Johannesburg.

Kamga, Morgan – School of Computational and Applied Mathematics, University of the Witwatersrand, Private Bag 3, Wits 2050, Johannesburg.

Keaikitse, Advice – Department of Mathematical Sciences, North West University, Mafikeng Campus, Private Bag X2046, Mabatho 2735.

Makanda, Gilbert – Department of Mathematics, University of Zimbabwe, Harare, Zimbabwe

Mambili-Mamboundou, Hermane – School of Computational and Applied Mathematics, University of the Witwatersrand, Private Bag 3, Wits 2050, Johannesburg.

Mavungu, Masiala – School of Computational and Applied Mathematics, University of the Witwatersrand, Private Bag 3, Wits 2050, Johannesburg.

Mberi Kimpolo, Charles – School of Computational and Applied Mathematics, University of the Witwatersrand, Private Bag 3, Wits 2050, Johannesburg.

Mhone, Peter – Department of Applied Mathematics, University of Limpopo, Private Bag X1106, Sovenga 0727.

Modhien, Naeemah – School of Computational and Applied Mathematics, University of the Witwatersrand, Private Bag 3, Wits 2050, Johannesburg.

Moretio, Thabo – Department of Mathematical Sciences, North-West University, Mafikeng Campus, Private Bag X2046, Mmabatho 2735.

Muatjetjeja, Ben – Department of Mathematical Sciences, North West University, Mafikeng Campus, Private Bag X2046, Mmabatho 2735.

Muchatibaya, Gift – Department of Mathematics and Applied Mathematics, University of Cape Town, Private Bag, Rondebosch 7701.

Ndaba, Cynthia – Department of Applied Mathematics, University of Limpopo, Private Bag X1106, Sovenga 0727.

Ntunka, Mbuyu – School of Chemical and Metallurgical Engineering, University of the Witwatersrand, Private Bag 3, Wits 2050, Johannesburg.

Okouma, Patrice – Department of Mathematics and Applied Mathematics, University of Cape Town, Private Bag, Rondebosch 7701.

Phillips, Colin – School of Computational and Applied Mathematics, University of the Witwatersrand, Private Bag 3, Wits 2050, Johannesburg.

Quagraine, Ekow – School of Computational and Applied Mathematics, University of the Witwatersrand, Private Bag 3, Wits 2050, Johannesburg.

Tchoualag, Laurent – Department of Mathematics and Applied Mathematics, University of Cape Town, Private Bag, Rondebosch 7701.

Tshehla, Sam – Military Academy, Saldanha.

Valla, Sahooda – School of Computational and Applied Mathematics, University of the Witwatersrand, Private Bag 3, Wits 2050, Johannesburg.

Wangra, Werhner – School of Computational and Applied Mathematics, University of the Witwatersrand, Private Bag 3, Wits 2050, Johannesburg.

Industry

Hansen, Joh – I.C. Consultants, P.O. Box 51190, Raedene 2124, Gauteng, South Africa

Hunter, William − Nissan South Africa.

Jekot, Tomasz − Super Group Supply Chain Partners.

Naismith, Alan − Mining Industry.

Pihlajasaari, Pekka − Nissan South Africa.

Povey, Stephen − Super Group Supply Chain Partners.

Slatter, Paul − Department of Civil Engineering, Cape Peninsula University of Technology, PO Box 1906, Bellville 7535.

Stacey, Richard − School of Mining Engineering, University of the Witwatersrand, Private Bag 3, Wits 2050, Johannesburg.

PROBLEMS

For each problem submitted by industry, the title of the problem, the industry presenting the problem, the industry representatives and the academic moderators are listed below.

Problem 1.

Title: The laminar-turbulent transition of yield stress fluids in large pipes
Industry: Mining
Industry Representative: Paul Slatter
Moderator: Tim Myers

Problem 2.

Title: Effect of thermal gradients on the random diffusion of a light solute in a heavier solvent
Industry: I.C. Consultants
Industry Representative: Joh Hansen
Moderator: Jean Charpin

Problem 3.

Title: Mine support mechanism using a limit equilibrium analysis
Industry: Mining
Industry Representative: John Napier
Moderator: David Mason

Problem 4.

Title: Multi-stage manufacturing sequence management
Industry: Nissan
Industry Representative: Pekka Pihlayasaari and William Hunter
Moderator: Colin Myburgh

Problem 5.

Title: Distribution and inventory cost optimization
Industry: Super Group Supply Chain Partners
Industry Representative: Tomasz Jekot and Stephen Povey
Moderator: Montaz Ali

Executive Summaries

A brief description of each problem is given followed by the equation-free Executive Summary for the problem

THE LAMINAR-TURBULENT TRANSITION OF YIELD STRESS FLUIDS IN LARGE PIPES

Mining Industry

Industry Representative
Paul Slatter, Department of Civil Engineering, Cape Peninsula University of Technology

Moderator
Tim Myers, Department of Mathematics and Applied Mathematics, University of Cape Town

Description

When yield stress fluids are transported through pipelines it is essential to maintain the stress throughout the majority of the pipe above the yield value. This prevents plug formation and so reduces the risk of blockage. The simplest way to achieve this is to keep the pressure drop sufficiently high that the flow is always turbulent. However, the higher the pressure drop the greater the cost of pumping and consequently operators attempt to maintain the velocity at a level only slightly above the critical velocity for turbulent flow. It is therefore crucial to be able to identify this critical velocity. This was the goal set for the study group.

Executive summary

For Newtonian flow the form of flow is determined by the Reynolds number, $\mathrm{Re} = \rho UL/\eta$. The transition to turbulence occurs at $\mathrm{Re} \approx 2100$. Unfortunately for non-Newtonian flow there is no clear definition of the Reynolds number. Experimental evidence indicates that the form

$$\mathrm{Re}_3 = \frac{8\rho V_a^2}{\tau_y + K\left(\frac{8V_a}{D_a}\right)^n},$$

provides the best correspondence. The various parameters in Re_3 are $\rho, V_a, \tau_y, K, D_a$ which represent the density, average annular velocity (i.e. in the flowing region around the central plug), yield stress, fluid consistency and width of the annular region, respectively. The annular velocity and annular width can be related to the average velocity, V, and pipe diameter, D. Given a value for the Reynolds number at which the flow becomes turbulent, this equation then provides

a relation between velocity and diameter. For large diameter pipes it is well known that the critical velocity is independent of the diameter. Re_3 is the only form of non-Newtonian Reynolds number that demonstrates this property.

A simple dimensional analysis indicates that for large $D \approx D_a$ the velocity takes the form

$$V = C(n)\sqrt{\frac{\tau_y}{\rho}}.$$

However, until now the value for the coefficient $C(n)$ had to be determined experimentally for a given fluid. Identifying $C(n)$ was the main aim for the Study Group activity.

Initially the group focused on determining the laminar velocity profile for a Herschel-Bulkley fluid flowing in a cylindrical pipe. This was achieved by two methods. Firstly a bi-viscosity model, where the viscosity in the central region is allowed to become infinite at the end of the calculation. Secondly a force balance was used. The velocity profile determines the average velocity in terms of the fluid parameters and the pipe diameter; it also provides expressions for the unknown parameters V_a, D_a in terms of V and D.

The key to solving this problem is the observation that for large diameter pipes, $D \to \infty$, the wall shear stress, τ_0, is approximately the same as the yield stress. Obviously it must be slightly larger, since otherwise the fluid would be a solid plug everywhere and no motion would take place. However, the analysis showed that in general $V \sim D(\tau_0 - \tau_y)^{n/(n+1)}$. Inspired by this, the group looked for an asymptotic expansion of the form

$$\tau_0 = \tau_y + \alpha_0 D^{-q} + \alpha_1 D^{-2q} + \cdots \qquad q > 0.$$

The condition that V is independent of D for large D requires $q = -(n+1)/n$. Substituting this expansion for τ_0 into the expressions for V and Re_3 led to a simple expression for the velocity (at leading order)

$$V = \frac{2n+1}{n+1}\sqrt{\frac{\mathrm{Re}_3}{8}}\sqrt{\frac{\tau_y}{\rho}} = C(n)\sqrt{\frac{\tau_y}{\rho}}.$$

This provides the exact form of the coefficient $C(n)$ in terms of n and Re_3. Theoretical results were shown to give a very close match to the experimental data provided.

Additional work was also carried out to determine the higher order correction terms in the relation for $V(D)$. The small diameter asymptote was identified and shown, at leading order, to take the form $V \sim D^{-n/(2-n)}$. By comparing the results of the analysis with experimental data we were able to show that the critical value of Re_3 above which the flow is turbulent is approximately the same as for the Newtonian problem (as suggested at the start of the meeting by Prof. Slatter).

EFFECT OF THERMAL GRADIENTS ON THE RANDOM DIFFUSION OF A LIGHT SOLUTE IN A HEAVIER SOLVENT

I.C. Consultants

Industry Representative
Joh O. Hansen, I. C. Consultants, P.O. Box 51190, Raedene 2124, Gauteng, South Africa.

Moderator
Jean Charpin, MACSI, Department of Mathematics and Statistics, University of Limerick, Limerick, Ireland.

Description
Crystals of high perfection are required in applications such as X-Ray optical systems and synchrotrons of high brilliance like the ESRF (European Synchrotron Radiation Facility). To meet the criteria of size and perfection required, crystals are grown using atom transport in an alloy of several metals. In the set-up presented at the Study Group, atoms are uniformly mixed in an alloy solvent with a height of two centimetres. A uniform thermal gradient is applied with the maximum temperature at the top of the solvent volume. Migration of atoms towards the colder end, in the direction opposite to buoyancy forces, is observed. This feeds the crystal growth that occurs at the bottom of the alloy. The Study Group was asked to explain and model the movement of atoms and to investigate the influence of operational parameters, particularly the temperature gradient.

Executive summary
In the current operating conditions, the movements of atoms are governed by two main phenomena, namely thermal diffusion, also known as the Soret effect or thermal migration, and Fick diffusion. Depending on the composition of the metal alloy, thermal diffusion will induce movement of atoms towards either the cooler or the hotter end of the solvent. The magnitude and direction of the migration in the alloy depends on a parameter known as the energy of transport. Simultaneously, Fick diffusion forces atoms to move from regions with high concentration towards regions where the concentration is lower. An equation describing these two effects was studied. The influence of ambient pressure and average temperature on the diffusion parameters was briefly investigated. The numerical results obtained with this model provided information regarding atom fluxes and the composition of the alloy.

Next, the effects of crystal growth on atoms were included in the model. Little consideration was given to the early stages of the growth, small crystals were assumed to be present at the bottom of the mixture right from the start of the process. The crystals act as atom sinks and their growth was described as such in the governing equation. At first the radial growth of each crystal was assumed constant. This approach proved over-simplistic and a more realistic model was suggested where the growth rate is proportional to the concentration of atoms that surround the crystals as well as the area of the crystal’s surface. The numerical algorithm developed during the Study Group will be extended to include this last model.

MINE SUPPORT MECHANISMS USING A LIMIT EQUILIBRIUM ANALYSIS

Mining Industry

Industry Representative
John Napier, School of Computational and Applied Mathematics, University of the Witwatersrand, Johannesburg

Moderator
David Mason, School of Computational and Applied Mathematics, University of the Witwatersrand, Johannesburg

Description

The analysis and design of support systems for mining excavations gives rise to interesting problems. Support systems include liner materials such as sprayed concrete, wire meshing and anchored strands of steel cable to contain the movement of broken rock near the surfaces of active excavations. The failed rock region near excavations and pillar edges cannot be assumed to behave elastically. An approximation that can be made is to represent the fractured rock as a pseudo-continuum in which the constitutive properties are assumed to be in a state of limit equilibrium. In this case a fixed constraint exists between the local stress components at each point of the material. A simple example of such a constraint is the imposition of the Mohr-Coulomb yield criterion.

The problem posed to the Study Group was to determine the detailed stress distribution and plastic flow movements in a specified region of interest that is bounded by a support lining material and to determine the effect of the lining material properties in controlling these movements.

Executive summary

An elastic-plastic theory was applied. A two-dimensional region of failed rock was considered. It was assumed that the equations of static equilibrium are satisfied in the failed rock region. The equations of static equilibrium consist of two equations for three components of the Cauchy stress tensor. The third equation was derived from the Mohr-Coulomb yield criterion. This criterion is a relation between the maximum and minimum principal stress components and the inherent shear strength of the rock. The Mohr-Coulomb yield criterion was expressed in terms of the components of the Cauchy stress tensor. The two-dimensional Airy stress function was introduced. The equations of static

equilibrium are identically satisfied and the Mohr-Coulomb yield criterion takes the form of a second order partial differential equation for the Airy stress function. By considering a scaling transformation a similarity transformation was derived and the Mohr-Coulomb yield criterion was rewritten as a second order ordinary differential equation in the similarity variable. The solution of this ordinary differential equation in a two dimensional wedge shaped region is being investigated.

Two extensions were also made of an existing one-dimensional limit equilibrium model to a two-dimensional region. The first model considered the shear stresses in the given form and then the average values of the normal components of the Cauchy stress tensor along the height of the region were obtained. This model can be considered as an approach to a plastic beam. A second term in addition to the one-dimensional term was derived The physical significance of this new term is being investigated. This model used the simplified Mohr-Coulomb yield criterion in the sense that the principal stresses were replaced by the normal stresses. To consider what the exact Mohr-Coulomb yield criterion can give a second extension of the one-dimensional model was introduced. The shear stresses in this model were of the same form as in the first model. The exact expressions for the normal stresses were obtained from the equilibrium equations. The collocation method was used to satisfy the yield criterion. A system consisting of ordinary differential equations and algebraic equations of the second order was obtained. A one term solution was found and investigated analytically.

MULTI-STAGE MANUFACTURING SEQUENCE MANAGEMENT

Nissan South Africa

Industry Representative
Pekka Pihlajasaari, Nissan South Africa

Moderator
Colin Myburg, School of Computational and Applied Mathematics, University of the Witwatersrand, Johannesburg.

Description

Automotive manufacturers are committed to producing many vehicle models on shared equipment, both to reduce investment in capital equipment and to allow rapid shifts in volumes to better react to market demand. These facilities are configured in multi-stage networks of conveyors with both merges and splits. Manpower is scheduled and facilities are configured to satisfy this demand subject to constraints imposed by different stages. Typical constraints include high switching costs between models, vehicles of the same colour must be grouped and combinations of options require different amounts of labour to fit. The vehicle production sequence needs to be chosen to best match capacity to demand while avoid the violation of facility and manpower constraints.

Executive summary

Since the problem attracted only a small number of participants, we decided to focus on a smaller but important subproblem. We chose the paint shop storage sequencing problem as the most amenable problem given the size of the Study Group and the time available. The paint shop storage area consists of several queues that store painted vehicles. The painting process disturbs the original production sequence and the manufacturer was interested in using the storage area to improve the sequence adherence. The group evaluated the current strategies for adding and removing a vehicle to and from the storage area and noted that there was some potential for process improvements. The current entry strategy simply places a vehicle at the tail of the first unfilled queue. The current exit strategy chooses a vehicle from the head of all the queues that adheres best to the planned production sequence. No production data was available, so the group decided to implement a simulator to generate planned production sequences with disturbances with specified statistical properties. Next the group implemented a framework for

testing various entry and exit strategies. Using the existing strategies as a baseline, various strategies proposed by the group could be evaluated with respect to improvement in production sequence adherence.

Reference

1. Pihlajasaari, P. Multi-stage manufacturing sequence management, December 2006. Unpublished.
2. Imada, O. New developments in milti-model mixed production and development process at motor vehicle manufacturers, Ritsumeikan University, May 2003.

DISTRIBUTION AND INVENTORY COST OPTIMIZATION

Supper Group Supply Chain Partners

Industry Representatives
Stephen Povey and Tomasz Jekot

Moderator
Montaz Ali, School of Computational and Applied Mathematics, University of the Witwatersrand, Johannesburg

Description

The problem involves the delivery of stock for replenishment in a selling station. It is assumed that depletion of stock follows a particular distribution, and hence he mean and the standard deviation of how the stock is depleted are known. When a level of L units is reached an order for a delivery of X units within a delivery time of T hours is placed with the distributor. The distributor has several modes of transport with known tariffs. The tariffs are calculated according to the delivery date and the amount transported, that is, the longer the delivery time and the more units ordered the lower cost per unit. Sales, ordering and delivery occur 24 hours a day, 7 days a week. The problem is to maximize the profit by choosing L, X and T per week given the price, cost and profit of an item of the stock.

Executive summary

A number of academics and students discussed the problem with the Industry Representatives over the first two days of the workshop. After understanding the problem the Study Group worked on the problem for another two days and came up with two different models: a deterministic and a stochastic model. At the final day of the workshop the two models were presented and the comments from the Industry Representatives were discussed. The comment was made that the deterministic model is a simplier version of the more complicated stochastic model. Because of the stochastic nature of the sale it was suggested that stochastic models need to be considered for a number of stages where the decision variables will be influenced by the stochasticity at each stage. This research is currently begin undertaken by a number of academics.

Detailed Technical Report